A history of science is not a textbook of science, but it should give us some clue to what is unfolding itself.

Finally, there is the question of omissions. Where the subject is so great and space so small it is a sorry task complaining that all has not been said, and of the special departments of science which fail to find a place we will mention only one—low-temperature research. We mention this because, apart altogether from personal prejudices and preoccupations, there can be no doubt that it is destined to be a major field of scientific activity, and one in which what has already been done, outstandingly important as it is, is almost certainly the prelude to developments of fundamental and revolutionary significance. But in speaking of omissions we have in mind not so much the landmarks of science itself as those events in its history which, while perhaps intrinsically of secondary importance, have nevertheless exercised an immeasurable influence on the course of history in general and the history of ideas in particular. Speaking of the intentional neglect of the "human sciences," Mr. Pledge writes: "To balance this, the story itself has been thrown against a background of the human and economic factors in successive periods rather than against the more intellectual and philosophical backgrounds which have mainly interested writers of previous general works." That is all very well, but there are limits to one's liberty to ignore important aspects of a movement by "throwing it against" a background which obscures them. A history of the Reformation which ignored the existing state of the Christian Church on the ground that the treatment was thrown against the background of the operations of the ductless glands would have some difficulty in defending its title: reformation of what? And a history of science which ignores the influence of science on knowledge in general meets with a similar difficulty. To take two examples: the discovery by Halley that comets were periodic and were not necessarily connected with the death of princes had an incalculable effect on the history of superstition, and the discovery by Adams and Leverrier of the planet Neptune by mathematical calculation had scarcely less influence on belief in the "reign of law." Not only are these effects ignored, but the discoveries themselves are not even mentioned. We are not without sympathy with the effort to give economic and material factors their due emphasis, but it is, after all, ideas that make history, and the profoundest influences of history are in the generation of new ideas.

But, when all is said, Mr. Pledge has given us a book of great value, and one for which there was great need. Space forbids us to mention its special characteristics of detail, but there are several novel features which reveal the seriousness with which the author has approached his task. We are indebted to him for a noteworthy addition to scientific literature and one to which we shall often refer. We may add a warning against judging the book, either qualitatively or quantitatively, by the price. It is a miracle of cheapness.

HERBERT DINGLE.

A Mathematician's Apology. By G. H. HARDY. (London: Cambridge University Press. 1940. Pp. vii + 93. Price 3s. 6d.)

The author of this little book is known to his professional colleagues throughout the world as one of the most eminent of living English pure mathematicians. To his many friends he is known as a man of strongly marked and highly original personality, which expresses itself very characteristically in his conversation. During the most anxious period of the dreadful summer of 1940 he spent some time in looking back, from the early evening of his life, on his own professional activities and in writing down his reflections on the nature and value of his subject and on his own attitude towards it. He took the manuscript somewhat diffidently to the University Press, thinking that they might perhaps consent to publish it as a pamphlet if he undertook to bear the expense. Mr. Roberts of the Press knows a good thing when he sees it, and he jumped at the opportunity to publish a fascinating and agreably written little book. I hope, and have good reason to believe, that his reward will be something more than a deferred annuity payable in the Kingdom of Heaven.

Professor Hardy distinguishes two questions: (1) What are the motives which lead certain persons to devote themselves to mathematics? And (2) What is the value of their activities?.

Professor Hardy's answer to his first question is as follows. The main motives which lead anyone of first-rate abilities to devote himself to any kind of research are intellectual curiosity, pleasure and pride in the successful exercise of his technical skill, and the desire for favourable recognition of his work by competent contemporaries and successors. An eminent mathematician has a particularly good chance of satisfying all these desires. Moreover, great mathematical gifts are so rarely accompanied by comparable ability in any other department that anyone who possesses them is under little temptation to aim at any alternative kind of achievement

In this connection there are two small points in Professor Hardy's argument which a captious reader might be inclined to criticize. (i) In his attempt to show that mathematical fame tends to be more enduring than, e.g. political fame, he does not sufficiently distinguish between continued admiration for a theorem in fact discovered by X and continued admiration for X as discoverer of that theorem. What he shows is that a first-rate mathematical theorem is likely to be recognized and admired as such by experts for an indefinitely long period. But surely it must depend on many highly contingent circumstances whether the name of its discoverer remains associated with it. Who of us, in admiring some theorem discovered by some Babylonian mathematician, is in a position to admire its discoverer for discovering it? No doubt there is a satisfaction in knowing that what is in fact one's work will continue to call forth admiration even though its admirers will cease to associate it with oneself. But that satisfaction might be enjoyed by many politicians and civil servants whose names will be unknown within a hundred years. (ii) Professor Hardy insists, probably with truth, that supreme achievement in mathematics is possible only at a comparatively early age. The only relevant instance which he gives is that of Newton. To produce, as he does, a list of persons who did supreme creative work in mathematics and then died young—e.g. Galois, Abel, Ramanujan and Riemann—is surely irrelevant. I suppose that the suppressed premiss is that the work which they did before their early deaths was so stupendously great that it is incredible that they should have equalled it if they had lived.

The discussion of the intrinsic value of mathematics is somewhat rambling, but the gist of it is as follows. A mathematical theorem is a pattern of ideas. (We are not told what special kind of ideas are the constituents of specifically methematical patterns.) If a theorem is to be non-trivial, it must have two characteristics, viz. beauty and seriousness. These are not independent, for the beauty depends to a considerable extent on the seriousness. Before attempting to analyse these characteristics Professor Hardy illustrates them by contrasting chess problems, which are genuine bits of mathematics but are essentially trivial and not particularly beautiful, with two very simple theorems which are both serious and beautiful. The examples which he takes are the proof that there is no greatest prime-number and the proof that there is no rational fraction whose square is equal to 2. These examples seem to me to be very happily chosen for his purpose; they are easy enough for any sane person to follow, and they are quite obviously weighty and beautiful. I think it would have been an advantage if Professor Hardy had distinguished more sharply between the proposition proved in a mathematical theorem and the reasoning by which it is established. In these two examples I should be inclined to feel that the beauty resides mainly in the reasoning and the seriousness mainly in the conclusion; but, no doubt, in more complicated examples each part would have a considerable share in both properties.

The seriousness of a theorem is said to depend on the significance of the ideas which it connects. This is said to depend in turn on a certain kind of generality and a certain kind of depth. A mathematical idea is general, in the sense required, if "it is a constituent in many mathematical constructs" and "is used in the proof of theorems of many different kinds." Professor Hardy does not profess to be able to give a satisfactory definition of "depth." From what he says I infer that the following statement would be a first approximation to what he has in mind. One theorem connecting certain ideas is "deeper" than another which connects the same ideas, if the former *cannot*, and the latter *can*, be proved without appealing to other ideas which are more complex or more general or more subtle. Mathematical beauty is said to depend on a combination of a high degree of unexpectedness with inevitability and economy. (I should suppose that the unexpectedness is in the conclusion, and the inevitability and economy in the proof.)

The value of mathematical activity of a high order is that it creates, or, to speak more strictly, disengages and reveals ideal patterns which have a high degree of beauty and significance, derived from the depth and the generality of their constituent ideas. It is thus essentially a form of artistic creation, which has the unique advantage that its materials are timeless concepts and not perishable sounds or pigments. He who demands some extrinsic justification for it betrays himself as a philistine; he who attempts to comply with such a demand is a disguised enemy or an uncomprehending friend.

The latter part of the book is devoted to showing that the parts of mathematics which are admirable for their beauty and seriousness are almost completely devoid of both utility and disutility. Here "utility" is taken to mean conduciveness to general safety, comfort, and happiness; and disutility is taken to mean its contrary opposite. The only parts of mathematics which can have appreciable utility or disutility are those which are applicable to the solution of practical problems of engineering, navigation, chemistry, physics, economics, etc. Now, Professor Hardy maintains, those parts of mathematics which have important practical applications are all intrinsically dull and trivial; whilst those which have great beauty and seriousness are without application to technical problems. This rule, he points out, is true of "applied" mathematics as well as "pure" mathematics. Applied mathematics is that part of the subject which is concerned with the formal structure of the actual physical world. Some of this is highly beautiful and serious, and its only defect is its limitation to that tiny region of the domain of formal possibilities which is exemplified by the actual world. But the beautiful and serious part of it is of no importance whatever to the technician.

I think that Professor Hardy is in the main right in these conclusions, but I also think that a very natural irritation with the aggressive philistinism of such a work as Professor Hogben's *Mathematics for the Million* leads him to exaggerate his case. Contemporary Communist writers bear a marked family likeness to the more bumptious and ignorant of the early nineteenth-century Benthamites, and the present reviewer in reading Professor Hogben's pronouncements on mathematics was constantly reminded of the famous Scottish utilitarian who described poetry as "the prodooction of a rude age." But this legitimate annoyance with self-satisfied ignorance and insensitiveness must not be allowed to cloud one's judgment. Surely it would be difficult to deny that Newton's theory of gravitation, Laplace's and Hamilton's reduction of the laws of dynamics to the Principle of Least Action, and Maxwell's theory of the electro-magnetic field are intrinsically beautiful and serious bits of mathematics. And surely they have had extremely important technical applications, both for good and for ill.

C. D. BROAD.

An Introduction to Hegel. By G. R. G. MURE. (Oxford: Humphrey Milford, at the Clarendon Press. 1940. Pp. xx + 180. Price 10s. 6d.)

It would seem that the days are gone when Hegel and the Hegelians could complain of not being understood, for here is another book that is thoroughly Hegelian in idiom, giving Hegel's own specific point of view rather than some Anglo-Hegelian version of it. The author has no intention of making any contribution to his subject; his concern is simply to expound Hegel correctly.

Unfortunately no attempt is made to explain the unique form of language used. As an introduction, the book would not introduce Hegel to a student that did not already know his philosophy fairly well. The author does not explain why he regards existing expositions of Hegel as unsatisfactory. In what way, for instance, does his book supersede the long introduction given by Professor Stace in his *The Philosophy of Hegel,* an exposition that is as intelligible to the beginner as we are ever likely to find in this sphere?

J. O. WISDOM.

David Hume: The Man and His Science of Man. Actualités Scientifiqus et Industriells, 860. By F. H. HEINEMANN. (Paris: Hermann et Cie. 1940. Pp. 67.)

This small essay has two parts. The first, by means of letters, some hitherto unpublished, and aided by psychoanalytic theory, the employment of which gives to it what novelty it possesses and endows the letters in question with an importance no one would otherwise assign to them, seeks to reveal Hume not as the ingenuous character of tradition but as a complicated personality suffering from internal conflict due to disappointments and frustrations demanding not only compensation but over-compensation. The conclusion drawn is that Hume's life was a *mixed* one, such as he affirmed in the *Enquiry* to be the most suitable to the human race. The second part deals with the idea of a science of man, in the discussion of which interesting points, left, however, undeveloped and indefinite, about Leibniz in relation to Hume are introduced, but the possibility of which, curiously enough, is made to rest on Hobbes and the sources of English philosophy, not on Hume, whose importance lies rather in his demand for a science of man and in the fact that he doubts statements which have been too easily taken for granted than in any great revelation about the nature of man. There is no attempt to elucidate what Hume had in mind when he declared that human nature enters into science as an important determinant and that a science of man